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## Inductive Single-Electron Transistor

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We demonstrate a sensitive method of charge detection based on radio-frequency readout of the Josephson inductance of a superconducting single-electron transistor. Charge sensitivity  $1.4 \times 10^{-4} e/\sqrt{\text{Hz}}$ , limited by a preamplifier, is achieved in an operation mode which takes advantage of the nonlinearity of the Josephson potential. Owing to reactive readout, our setup has more than 2 orders of magnitude lower dissipation than the existing method of radio-frequency electrometry. With an optimized sample, we expect uncoupled energy sensitivity below  $\hbar$  in the same experimental scheme.

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Quantum measurement in the solid state has been shown to be feasible as several realizations of qubits based on mesoscopic superconducting tunnel junctions have emerged [1–4]. Measurement of physical quantities close to the limit set by the uncertainty principle is, on the other hand, an important issue in its own right. The detector should not only have a high gain, but also its internal noise should not act back on the measured observable more than allowed by the uncertainty principle.

The single-electron transistor (SET) is a basic mesoscopic detector, sensitive to electric charge on a gate capacitor. Its operation is based on stochastic tunneling of single electrons, typically at sub-Kelvin temperatures  $k_B T \ll e^2/(2C_\Sigma)$ , where  $e$  is the electron charge and  $C_\Sigma$  is the total capacitance of the SET. Because of shot noise the SET has, however, a quite substantial backaction, and even theoretically, its energy sensitivity [5] remains approximately a factor of 4 from the fundamental quantum limit  $\hbar/2$ .

The SET has a practical problem with a low bandwidth, but a few years ago it was demonstrated [6] that a band of tens of MHz is possible with radio-frequency SET (rf-SET) where impedance match from a high-impedance SET to  $50 \Omega$  is achieved by means of an  $LC$  tank circuit. Several picowatts of power is dissipated in the rf-SET under optimal working conditions, which warm up the SET island up to half a Kelvin.

Cooper pair tunneling, in contrast, is correlated, and hence a detector based solely on the Josephson effect does not exhibit shot noise or dissipation. Zorin has introduced a superconducting quantum-limited low-frequency electrometer [7,8], and a theory for a corresponding high-frequency device [9] where readout is performed similarly as in the rf-SQUID (see also [10]). Any experimental demonstrations, nevertheless, of nondissipative high-frequency electrometry have been lacking to date.

In this Letter we present a new type of such a device, the inductive single-electron transistor (L-SET), and demonstrate it in experiment. Two distinct operation modes are identified. The “plasma oscillation” mode corresponds to small oscillations of the Josephson phase

$\varphi$  in harmonic potential, whereas in “anharmonic” mode, higher excitation is used such that the swing is more than  $2\pi$ . Charge sensitivities of  $2.0 \times 10^{-3} e/\sqrt{\text{Hz}}$  and  $1.4 \times 10^{-4} e/\sqrt{\text{Hz}}$  were achieved in the two modes, respectively. The present experiment is also the first one to directly probe the nonlinear dynamics of a mesoscopic superconducting junction, in terms of both the amplitude and phase of microwave voltage reflection coefficient  $\Gamma$ .

A superconducting SET (SSET, also called Cooper-pair transistor) has two small-capacitance Josephson junctions in series, and the Hamiltonian is  $(q - q_g)^2/(2C_\Sigma) - 2E_J \cos(\varphi/2) \cos(\theta)$ . Here,  $q_g$  is the gate charge,  $\varphi$  is the phase difference across the whole device, assumed to be a classical variable due to an environment having impedance much smaller than  $R_Q = h/(2e)^2 \simeq 6.5 \text{ k}\Omega$ , whereas  $\theta$ , conjugate to the charge of the island  $q$ , experiences quantum effects. Evidence of the resulting band structure  $E_n(\varphi, q_g)$  has been obtained in experiment [11]. The lowest band  $n = 0$  has approximately sinusoidal energy  $E_0(\varphi, q_g)$ , and thus, the SSET is effectively a gate-tunable single junction [12]. From our point of view the most important property of SSET is its Josephson inductance  $L_J^{-1}(\varphi, q_g) = (2\pi/\Phi_0)^2 \partial^2 E(\varphi, q_g)/\partial \varphi^2$ , where  $\Phi_0 = h/(2e)$  is the flux quantum. With a total shunting capacitance  $C$ , a SSET thus forms a harmonic oscillator with the plasma frequency  $f_p = 1/(2\pi)(L_J C)^{-1/2}$  for small  $\varphi$ .

In our experiment,  $f_p$  is tuned into an experimentally accessible range, below 1 GHz, by a large  $C \simeq 8.4 \text{ pF}$ . The SSET is also shunted with an external inductor  $L \simeq 7.4 \text{ nH}$  which enables the anharmonic mode (see later), and also practically eliminated sample breakdown due to accumulating static electricity during preparations. The circuit (see Fig. 1) is coupled to a feedline via a coupling capacitor  $C_c \ll C$ . The sources of dissipation are lumped into the resistor  $r$  in series with the SSET.

Energy of the system of the SSET plus the  $LC$  oscillator is the sum of potential energy  $E_n(\varphi, q_g) + \Phi^2/(2L)$ , approximately  $-E_0(q_g) \cos(\varphi) + [\Phi_0^2/(8\pi^2 L)]\varphi^2$  at the ground band, and kinetic energy  $q^2/2C_\Sigma$ . Here,  $\Phi$  is the

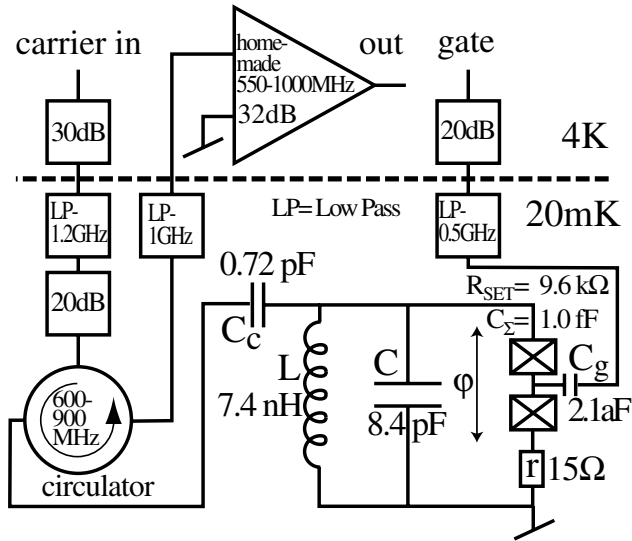


FIG. 1. Schematics of the L-SET circuitry. A SSET is coupled in parallel with an  $LC$  circuit resonant at  $f_0 = 613$  MHz.

magnetic flux in the loop. Assuming the SSET stays at the lowest band, where  $I \approx I_0 \sin(\varphi)$ , classical dynamics of phase in the oscillator is thus analogous to a particle moving in a sinusoidally modulated parabolic potential. The dynamics is similar to that in the rf-SQUID [13], i.e., a single junction shunted with a loop inductance, except that since our loop is not fully superconducting, flux quantization or flux jumps do not exist. At small driving amplitude (linear regime), the phase particle experiences harmonic oscillations around  $\varphi = 0$  at the frequency  $f_p = 1/(2\pi)(L \parallel L_J C)^{-1/2}$  which is controlled by gate tuning  $L_J$ . This mode of operation, where the L-SET works as a charge-to-frequency transducer, we call the “plasma oscillation mode.”

At higher oscillation amplitude, the particle sees a different local curvature of the potential due to the  $\cos(\varphi)$  term, thus changing the oscillation frequency. At very high amplitude, the cosine modulation becomes effectively averaged out. Thus, when increasing excitation, we expect a change of resonant frequency from  $f_p$  to  $f_0 = 1/(2\pi)(LC)^{-1/2}$ . This change of resonance frequency at a critical excitation power  $P_c$  can be seen as an analog of a dc-biased Josephson junction switching into a voltage state [14]. Qualitatively, this type of behavior was identified in the experiment.

At large excitations, the highly nonlinear oscillator experiences complicated dynamics which do not in general allow analytical solutions. Numerical calculations over a large range of parameters, however, show consistently that the system response depends on  $L_J$  also in this case [15]. This is what we call the anharmonic mode.

The Al SSET was of a standard design, incorporating Cu quasiparticle traps  $4 \mu\text{m}$  from the junctions, positioned such that they are not on the conduction path. From the total tunnel resistance  $R_{\text{SET}} = 9.6 \text{ k}\Omega$  we

get the single-junction Josephson energy  $E_J = 1.56 \text{ K}$ . Surface area of the two tunnel junctions  $0.021 \mu\text{m}^2$  gives an estimate of the charging energy of the SET (capacitance  $\sim 43 \text{ fF}/\mu\text{m}^2$ ),  $E_C \approx 1.0 \text{ K}$ . However, this method does not allow sufficient accuracy due to substantial sensitivity of the electrometer performance to  $E_J/E_C$ . The best fit was obtained with  $E_J/E_C = 1.7$ , thus  $E_C = 0.92 \text{ K}$  and  $C_\Sigma = 1.0 \text{ fF}$ , which is within error margins of the previous method. The value of each tank circuit component was determined prior to cooldown to roughly 20% accuracy. These agreed with values gotten by fitting to frequency responses.

The measurements were done in a dilution refrigerator at a base temperature of  $20 \text{ mK}$ . Every effort was made to eliminate noise. The microwave feedline was heavily attenuated and low pass filtered. A circulator at  $20 \text{ mK}$ , tested to have more than  $20 \text{ dB}$  backward isolation, was used to cut backaction noise from the cold amplifier. The coaxial gate lead was filtered and attenuated at  $4 \text{ K}$ , and it had small capacitance  $C_g \approx 2.1 \text{ aF}$  in order to avoid voltage noise.

We used microwave reflection to read the reactance of the oscillator:  $\Gamma = (Z - Z_0)/(Z + Z_0)$ , where  $Z_0 = 50 \Omega$  and  $Z$  is the sample impedance, was measured by probing the system with a carrier wave, typically  $\leq -115 \text{ dBm}$ . The reflected carrier was amplified with a chain of amplifiers having a total of  $5 \text{ K}$  noise temperature and detected with a network analyzer or a spectrum analyzer.

At low excitation, we measured the plasma resonance at  $f_p = 723 \text{ MHz}$  and its shift with gate of  $15 \text{ MHz}$  (see Fig. 2). With the best-fit tank circuit parameters, the shift corresponds to a 15% modulation of  $L_J$ , achieved according to theory at  $E_J/E_C = 1.7$ , in good agreement with independently determined SSET parameters. For this  $E_J/E_C$  we calculate the minimum (with respect to  $q_g$ ) value of the SSET inductance  $L_J = 15.4 \text{ nH}$ . Placed in parallel with  $L$ , we thus expect  $f_p = 744 \text{ MHz}$ , which agrees rather well with the experimental value. The small disagreement can be due to uncertainty in the values of  $L$  and  $C$ .

The frequency response was periodic with respect to gate voltage with a perfect  $2e$  period. By increasing temperature, we observed a transition to full  $e$  periodicity at  $300 \text{ mK}$  as usually in a SSET.  $Q_e$ , determined as the half-width of the resonance, reduced from 18 to 12 [16] while tuning by gate from the maximum to minimum in  $f_p$ , which is presently not understood.

As the excitation is increased, the plasma resonance ends at  $P_c = -116 \text{ dBm}$  (with numerical simulations, not shown, we got a similar value), as qualitatively expected from the simple reasoning based on the potential energy. In Fig. 3, this effect is seen as a rather abrupt switch of the broad plasma resonance, centered at  $720 \text{ MHz}$ , into a narrower tank resonance at  $613 \text{ MHz}$ . The wavelike texture at  $-105 \dots -90 \text{ dBm}$  is due to the  $\cos(\varphi)$  Josephson term. Changes in coupling, e.g., that  $Z$  goes

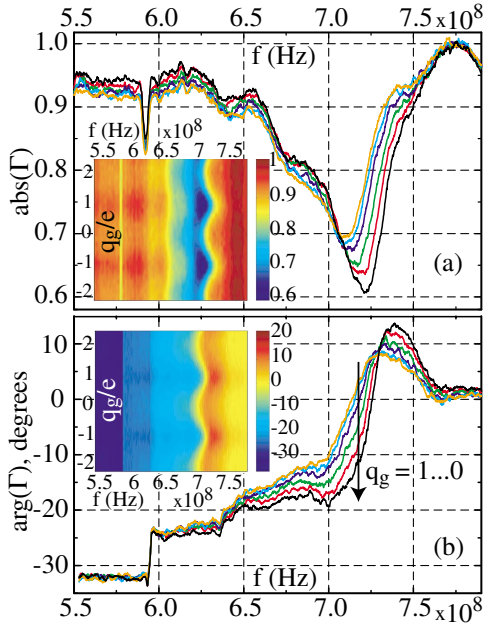


FIG. 2 (color online). Measured frequency response at  $-125$  dBm for successive gate values  $q_g/e = 1 \dots 0$ , (a) amplitude, (b) phase. Effective resolution bandwidth  $\approx 1$  Hz. The insets portray the periodicity.

through critical coupling at  $-105$  dBm, and sharpening of the resonance are related to an increase of  $Q_i$  from 20 below  $P_c$ , up to several hundreds. Since  $Q_i = \omega_0 L_J / r$ , and since  $L_J$  acquires a large effective value at a high drive  $\gg 2\pi$  due to canceling of positive and negative contributions, the supposed dissipation of  $r$  in series with the SSET has less effect at  $P \gg P_c$ . The satellite dips, which did not move noticeably with respect to gate dc level, power, or temperature,  $\pm 19$  MHz around the tank circuit resonance arise probably from a spurious low-frequency resonance that couples to the system.

The only internal sources of noise and backaction are fluctuations in the resistor. At 20 mK these are already at

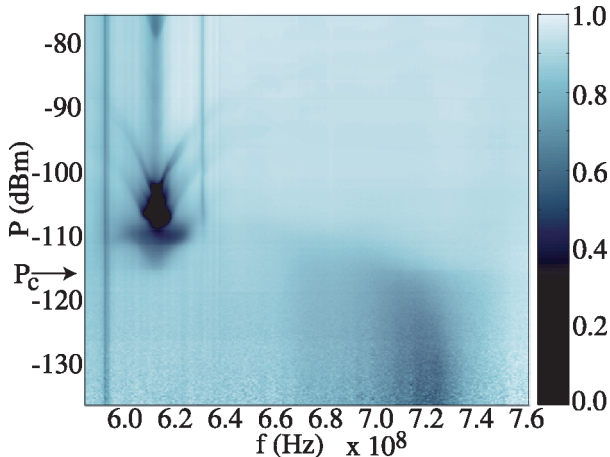


FIG. 3 (color online). Measured frequency response ( $|\Gamma|$ ) as a function of excitation power. “Critical power”  $P_c$  (see text) is marked.

the quantum limit  $\hbar\omega \approx k_B T$ . Although the cold amplifier has a noise temperature  $T_N \gg T$ , temperature in the sample is only approximately doubled to 40 mK by leakage through the isolator. Noise in the output, however, is fully dominated by the amplifiers. This is converted into an equivalent charge sensitivity  $s_q$  or energy sensitivity  $\epsilon = s_q^2 / (2C_\Sigma)$  by  $s_q = s_V / (\partial V / \partial q)$ , where  $s_V = \sqrt{Z_0 k_B T_N}$  in amplitude readout of the reflected carrier voltage  $V = V_0 |\Gamma|$ . In readout of the phase  $\phi = \arg(\Gamma)$  we have  $s_q = s_\phi / (\partial \phi / \partial q)$ , where  $s_\phi = \sqrt{2} s_V / V$ .

The derivatives can be written as  $\partial V / \partial q = V_0 (\partial |\Gamma| / \partial l_J) (\partial l_J / \partial q)$ , where  $l_J = L_J / \min(L_J)$ , and similarly for the phase. In the plasma oscillation mode,  $\partial |\Gamma| / \partial l_J$  is computed by standard circuit analysis, but in the anharmonic mode, we rely on numerical simulations. The “gain”  $g = \max(\partial l_J / \partial q)_{q_g} e$ , expressed as relative modulation of the Josephson inductance per electron, is the main figure of merit of the SSET electrometer,  $g = 0.23$  for the present sample according to the theory (Fig. 4).

In the plasma oscillation mode, the optimal excitation ( $P_c$ ) corresponds to critical ac current. For the present sample we calculate in detail [15], both in amplitude or phase readout,  $s_q = 5 \times 10^{-4} e / \sqrt{\text{Hz}}$ . In the anharmonic mode,  $\partial |\Gamma| / \partial l_J$  is smaller according to simulations, but the sensitivity is predicted to be better by an order of magnitude,  $s_q \approx 4 \times 10^{-5} e / \sqrt{\text{Hz}}$ , due to the possibility for a high excitation.

Charge sensitivity was measured in a straightforward manner using amplitude readout. A small  $1/20 e_{\text{rms}}$  marker signal was fed to the gate at a frequency  $f_g$ . The modulated carrier was detected with a spectrum analyzer, and signal-to-noise ratio was read from sidebands at  $f_p \pm f_g$ . In the plasma oscillation mode we got  $s_q = 2.0 \times 10^{-3} e / \sqrt{\text{Hz}}$  at the maximum power

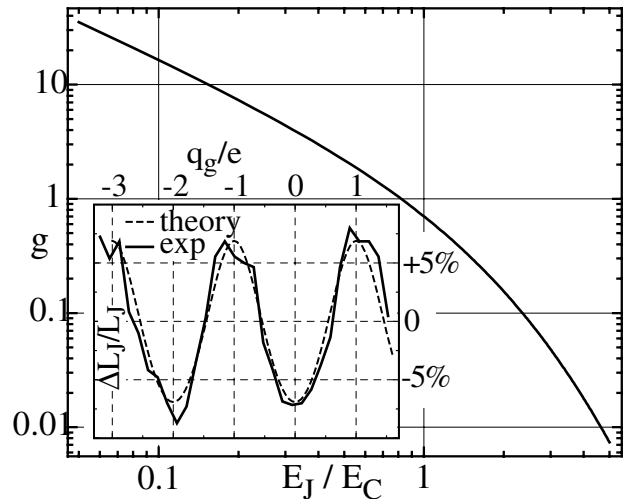


FIG. 4. Electrometer gain  $g$  calculated as a function of  $E_J / E_C$ . The inset shows gate modulation of  $L_J$  for the present sample.

$P_c \approx -116$  dBm, corresponding to 20 fW dissipation in the whole resonator circuit. Note that due to Cooper pair tunneling, the power is not dissipated in the SSET island. In the anharmonic mode, significantly better sensitivities were obtained.  $s_q = 1.4 \times 10^{-4} e/\sqrt{\text{Hz}}$  was measured at  $-100$  dBm [17]. Because of a higher impedance mismatch, power dissipation can be kept below 20 fW in this case as well.

The sensitivities were measured at  $f_g = 300$  kHz, however, 3 dB bandwidths  $\Delta f$  were roughly 40 MHz in the plasma oscillation mode and 10 MHz in the anharmonic mode, in agreement with inverse response time of the oscillator,  $\Delta f \approx f_0/Q_e$ .

The discrepancy of a factor of 4 in the predicted sensitivities is partially explained through another process not included in the model. Namely, according to the simple model, inverse sensitivity should increase linearly with amplitude. A clearly weaker dependence was found in the experiment, however. At small amplitude, theory, and experiment are within a factor of 2 from each other. We attribute the effect to an increased rate of Zener transitions to a higher band when the phase starts to reach the minimal band gap of the SSET at  $\varphi = \pm\pi$ . At the second band  $|L_J| \gg L$  and thus it has little effect on the resonant frequency. Experimentally, close to  $P_c$ , both resonances appear simultaneously, in agreement with the SSET being at the second band for part of the time.

As seen in Fig. 4, the gain grows roughly as  $(E_J/E_C)^{-1}$  at small  $E_J/E_C$ .  $\partial|\Gamma|/\partial l_J$  is a rather weak function of the parameters of the sample or the tank circuit; however, small total inductance and matching to  $Z_0$  are favored. For low  $E_J/E_C$ , difficulties arise due to noise sensitivity. Let us evaluate the ultimate performance of the electrometer for  $E_J/E_C = 0.15$  which we consider as still usable with our present noise level.

The anharmonic mode offers good possibilities to reach uncoupled energy sensitivity of  $\epsilon \approx \hbar$  in the present configuration, using an Al SSET. Our simulations indicate that  $\partial|\Gamma|/\partial l_J$  is insensitive to  $L_J$ . With the “optimal” Al SSET,  $R_{\text{SET}} = 35$  k $\Omega$ ,  $E_J = 0.45$  K, and  $L_J = 60$  nH, we have a prediction of  $s_q \approx 3 \times 10^{-7} e/\sqrt{\text{Hz}}$ . Since the minimal band gap between the 1st and 2nd bands in a SSET (in contrast to a Cooper-pair box for instance) is insensitive to  $E_J/E_C$ , the decrease of sensitivity due to mainly interband transitions amounts by roughly the same factor  $\eta \approx 4$  as in the present sample, thus we expect to be feasible  $\eta \times s_q \approx 1 \times 10^{-6} e/\sqrt{\text{Hz}}$ , corresponding to  $\epsilon \approx \hbar$ .

The best prospects to reach a quantum-limited operation in the plasma oscillation mode with  $Q_i \leq 20$  are expected if materials with higher  $T_c$  than Al are used. Using a rather standard Nb-Al technique [18], with “effective”  $\Delta = \sqrt{\Delta_{\text{Al}}\Delta_{\text{Nb}}} \approx 0.5$  mV,  $\eta \times s_q \approx 1 \cdot \cdot \cdot 2 \times 10^{-6} e/\sqrt{\text{Hz}}$  if a SQUID preamplifier with  $T_N = 0.3$  K,

which offers an additional benefit of a lower backaction and thus less stringent conditions for noise isolation, is used. In the anharmonic mode, equally good sensitivities are simulated for Nb-Al as for the case of Al.

Finally, we note that the system may provide a model system of a qubit plus an integrated oscillator which works also as a detector [19], with an inherent advantage of efficient filtering of external noise outside the band of the resonator.

In conclusion, we have demonstrated the feasibility of the L-SET, where we use gate-dependent Josephson inductance of a superconducting single-electron transistor for radio-frequency electrometry. We foresee excellent prospects to operation close to the quantum limit.

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